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BONAVENTURA CAVALIERI

da una stampa antica

THE MATHEMATICS TEACHER

Volume XXV Number 2

Edited by William David Reeve

Understanding and Mechanical Performance in Algebra*

By E. R. BRESLICH

School of Education, University of Chicago

EVERY new study of the results obtained by pupils in algebra adds further evidence to the findings of former studies that pupils master surprisingly little of the subject matter and processes taught in that course. Teachers of mathematics have not been indifferent to this problem. Efforts to solve it have been in the direction of giving an increasing amount of attention to such measures as supervision of pupils at study, careful rationalization of concepts and processes, development of generalization, provision for individual differences, and scientific administration of practice and drill. However, they seem to have accomplished little toward correcting the situation.

The problem is difficult to solve on account of its complexity. For the teaching of algebra has a two-fold aim. Through the study of the subject the pupil is to be trained to think clearly, to draw correct inferences, to develop and use good judgment, and to make fine discriminations. At the same time, he is to become proficient in the mechanical processes of algebra and to acquire the ability to perform them correctly when they occur in problem situations.

* An address delivered before the Mathematics Section of the Kansas State Teachers Association, November 5, 1931.

Many devices have been invented to improve the mechanical phases of algebra. Practice exercises, instructional tests, drill cards and workbooks are recommended as means of preventing forgetting and of making retention permanent. In many schools these aids have become an essential feature of instruction. Nevertheless, when the pupils are tested some time after they have finished the course in algebra or when they are called upon in other work to use what they have been taught they show invariably a lack of ability even in the simplest manipulative processes. When test items require reasoning and real understanding the results are still more discouraging. Indeed, some investigators have interpreted these findings to mean that too much emphasis on the mechanics of algebra has the tendency to discourage logical thinking and to keep the pupils from understanding the true meaning of the subject.

When pupils employ without understanding such phrases as canceling, transposing, clearing of fractions, and removing parentheses, their work amounts to nothing more than meaningless juggling of symbols and guessing answers. Although many teachers place their faith entirely on mechanical drill and regard as lost the time spent on rationalizing and on other procedures aiming to lead the pupil to understanding, such work may in fact be a source of economy and an essential step toward mastery. If the teacher succeeds through mere drill work to train pupils to make rapid responses that are correct in certain drill situations it does not follow that the pupils will know what to do when the same processes occur in other more complex situations. Indeed, tests show that a slight change from one simple situation to another has a marked effect on the correctness of the response. In a recent study[1] it was found that only half as many pupils who responded correctly to the problem $8.6a+5.3a-7.02a=$, were able to respond correctly to the problem: Add $3.48x$ and $-5.2x$.

The method of developing manipulation exclusively does not automatically develop insight and understanding. Thus, a pupil might solve correctly $4x+2=14$, but fail with $3k=8+k$, although the principles employed in both cases are the same. Intensive drill might enable pupils to make correct responses in certain problems without

[1] H. E. Benz, *The Learning and Teaching of Algebra*. Columbus, Ohio: The State Department of Education, 1930.

an understanding of the meaning of the symbols or of the nature of the processes to be performed, but pupils so trained fail to derive the real educational values from the study of the subject.

A pupil may give the correct answer to a problem, but the teacher cannot always be sure that he understands it. For example, a test given by the writer contained the questions $(ab)^2=$———— and $(a+b)^2=$————. Some pupils responded correctly to both questions. Others gave the correct result, a^2b^2, for the first and the incorrect result a^2+b^2, for the second. When they explained their solutions to the teacher it was found that they arrived at the results in different ways. The procedure of those who answered both questions right seemed to be as follows: They quickly examined the problem, caught its meaning, and thought out the result. The others merely wrote the exponent *2* over each of the letters *a* and *b*. They expressed surprise that the method should give the right answer in the first problem and fail in the second. The fact was that no real learning had taken place, and strictly speaking, the apparently correct result for the first problem was wrong as truly as the incorrect result for the second. Pupils who carry out their manipulations without understanding of what they mean are often thrown into a state of confusion because in one case the answer is marked right while in another it is wrong. It is indeed unfortunate that the mechanical way occasionally gives the correct result. Thus, in multiplication it is found that pupils reacting automatically to the sign of operation find the problem $\frac{a}{b}\times\frac{c}{d}$ easy. If they have $\frac{ac}{bd}$ for an answer the work is marked right. However, when they use the method which gave $\frac{ac}{bd}$ on $\frac{a}{b}+\frac{c}{d}$ they fail. The probability is that they know no more about multiplication than about addition although they scored high in the test on multiplication.

A third problem in the foregoing test was to solve the equation $v=bh$ for h. Several pupils gave wrong answers as $v-b$, $\frac{b}{v}$ and $b-v$. In the conference which followed the marking of the papers it was disclosed that the pupils mechanically "moved," "transposed," or "brought over" the b from one side of the equation to the other.

That was all they could see in the problem. They did not know the principle on which the process was based. However, when the pupils who had given the right answer, $\frac{v}{b}$, were questioned, it was found that some had also arrived at the correct result by the mechanical, although erroneous method. They merely happened to make a lucky guess. They were just as much in need of reteaching as the others. The problems had been marked right, but no real learning had taken place.

A fourth problem in the test was $(x^2)^3 =$ ———. Here the most common wrong answer was x^5. Some pupils gave x^8. Both answers displayed a lack of understanding of the meaning of exponents. However, some of those who gave the correct answer, x^6, volunteered to change it to x^5 when pressed for an explanation.

Mention was made earlier of the poor showing made by pupils who have studied algebra. They seem to give evidence of mastery of very little, and if a careful analysis is made of the knowledge of the pupils who respond correctly in the tests that little is reduced even further. It is apparent that the devices which in recent years have been introduced for developing speed in algebraic computation do not alone produce lasting results in mechanical algebra, and certainly not in situations which require understanding and careful discrimination. Until the efficiency and value of these devices have been established, teachers should not put too much faith in them. However, when devices are used to develop manipulative proficiency the teacher should secure evidence that understanding has been attained before drill begins. To help the pupil acquire this understanding three important facts have proved to be exceedingly helpful and should be kept in mind by the teacher: First, the pupil should receive teaching and reteaching until he knows the meaning of the fundamental concepts of algebra which he employs in the manipulative processes. This refers to such concepts as literal number, signed number, exponent, equation, fraction, and ratio. Second, the pupil should constantly receive training in analyzing the relationships expressed by the algebraic symbols. Third, as much as possible he should discover independently the short cuts which facilitate manipulation.

Two methods are usually employed in developing meanings of new concepts. One gives the definition and then provides examples and

experiences to clarify the meaning of the definition. The other provides the experiences first and lets the meaning of the concept grow out of them. Observation of teaching shows that the first method is likely to become purely mechanical. It might amount to no more than the memorization of meaningless facts supplemented by frequent drill in stating the definitions. It is difficult to know whether real understanding has been attained. The second method aims at understanding first. Gradually the learner is made acquainted with the essential characteristics and properties of the term. He is soon able to give a good description of the term in his own words and shows that a clear notion has been acquired. His formulation is then refined by the teacher and the definition is given. The first method proceeds rapidly. It simply states that $a \times a$ is to be written briefly a^2, $a \times a \times a = a^3$, $a \times a \times a \times a = a^4$, and $a \times a \times a$ to n factors $= a^n$. It seems very easy, and the pupils assure the teacher that they understand. However, diagnosis of written work as to errors and the confusion in later work give evidence that much of the difficulty may be traced back to a lack of understanding of the exponential notation.

For example, the meaning of "exponent" may be developed as follows: The problem of measuring the surface of a square is proposed. By measuring surfaces of squares whose sides are 2 inches, 3 inches, etc., the results, 2×2, 3×3, etc., are obtained. Likewise, the area of a square whose side is a inches would be $a \times a$. The pupil is then introduced to the brief way of writing the product of two equal factors and changes 2×2, 3×3, . . . $a \times a$, to 2^2, 3^2 . . . a^2. Finally, the formula $A = a^2$ is developed and practice is given with problems relating to areas and squares. Such work may be done in the seventh grade and considerable time should be devoted to it to enable all pupils to associate the exponent 2 with the product of two equal factors. The repeated association with the square makes the terminology seem rational to the learner.

The next step is to measure the interior of a cube. Volumes of cubes are found to be $2 \times 2 \times 2$, $3 \times 3 \times 3$, . . . and $a \times a \times a$. As in the case of the square, the products are abbreviated to 2^3, 3^3, . . . and a^3. Since second- and third-degree expressions satisfy for some time to come the pupil's algebraic needs there is no reason for passing on to higher powers at the present. The generalization from a^2, a^3, to a^4, a^5, and a^n is not easy and should be left to the time when need for it arises. One strong feature of the foregoing method is that it pro-

ceeds slowly and allows the pupil ample time to assimilate new ideas.

Emphasis on the relationships involved in algebraic situations is a useful device for helping pupils acquire insight and understanding. To illustrate, one of the common mistakes in the problem $(3x - 4) + (6x - 3) =$ ——— is to change the signs of $6x$ and -3 and to add the result to $3x - 4$. When pupils are asked to explain the solution of the problem they readily state that they are "removing parentheses." They fail to pay any attention to the plus sign, but upon recognition of the parentheses they immediately proceed to "remove" it by changing the signs of the terms. Apparently in the teaching of "removal of parentheses" too much emphasis is being placed on the removal of parentheses when they are preceded by a minus sign, which is natural enough because it is a difficult case. The large amount of practice establishes the habit of changing signs in parentheses problems, and later the pupils change signs even when the parentheses is preceded by a plus sign. Experience has shown that more emphasis on the indicated operation and less on purely mechanical performance goes far toward eliminating or reducing that particular error.

The term "cancel" is responsible for a variety of mistakes. To most pupils it seems to mean nothing more than the crossing out of something. "I can cancel in $\frac{3a}{b} \times \frac{5}{6a}$," is a common explanation, when the pupil is asked to tell how he worked the problem, "why not in $\frac{3a}{b} + \frac{5}{6a}$, or in $\frac{3a}{b} = \frac{5}{6a}$?" With him it is not a matter of recognizing the relationships or of performing the indicated operations. He seems to recognize in a hazy way the general form on which he was drilled when he was taught to cancel. Disregarding the symbol of operation, he proceeds to cross out certain letters. To reduce this mistake let the teacher insist that the learner classify every exercise in regard to the operation indicated by the symbols. The foregoing examples are to be thought of as exercises in multiplying, adding, or solving an equation, not in canceling.

Another term which brings much grief to the pupil of algebra is "transposition." Like "removing parentheses" and "canceling" its meaning is usually not understood. Every study of errors in alge-

braic work contains the evidence. In a test given by the writer a considerable number of pupils solved the equation $3x=8+x$ as follows:

$$\begin{aligned} 3x&=8+x \\ 4x&=8 \\ x&=2 \end{aligned}$$

Some presented the following solution:

$$\begin{aligned} 3x&=8+x \\ 4x&=8 \\ x&=4 \end{aligned}$$

Other wrong answers were $5x$, 8 and 5. Evidently the pupils presenting such solutions had no idea of the relationships involved. They "transposed" the x but forgot about "changing the sign." They "transposed the 4 by subtracting it from 8 without giving thought to the relationships involved in the statement. It seems advisable not to teach the mechanical process of transposition at all but to place much emphasis in teaching on the relationships between the algebraic symbols, and on the fact that any indicated process should be undone by the use of the inverse process. Thus, to undo the addition in $8+x$ the pupil should use a subtraction. To undo the multiplication in $4x$ he should divide. With such emphasis on relationships it is possible to develop understanding to the extent that pupils master the method of solving any equation of the form $ax=b$. To pupils thus trained the equations $.05x=8$, $2\pi r=8$, and $ax=8$ are no more difficult to solve than $4x=8$.

The third of the foregoing suggestions for developing understanding in algebra is to lead the pupil to discover independently the laws of algebra. For example, he should work problems like $a^2\times a^3$ by counting mentally the number of equal factors. Soon he will discover a quick way of getting the result. He will then formulate the law of multiplication. Teachers often make the mistake of not allowing him enough time to do this. Two or three examples are given and the teacher states the law. From that point on the pupil merely adds exponents. If the character of a problem is slightly changed, as in $a^2\times b^3$, the pupil finds himself unable to decide what to do. Instead of deciding his next step on the basis of the meaning of exponents he makes guesses, such as a^5b, or ab^5, or perhaps a^5b^5. Evidently this

method of teaching exponents is unsafe, because the law of exponents is introduced and used before it is really understood. What is needed is repeated returning to the meaning of exponents in a large variety of problems, until the law is discovered. Even after the law has been stated the pupil should frequently check his results by counting equal factors.

The foregoing discussion should not be interpreted to mean that all mechanical devices in the teaching of algebra should be discarded, but rather that when they are used they should be the outcome of the pupils' experiences and should be discovered by him under the careful guidance of the teacher. A typical example of this type of instruction is the use of the table in teaching pupils to solve verbal problems. They should at first write all the facts in full. For example, in a motion problem such statements as the following may be made:

Let x be the number of miles traveled by the first train.

Then $x+6\frac{1}{2}$ is the number of miles traveled by the second train. Other statements follow until all the facts are recorded. Sooner or later the pupil should be led to discover that he may save work and time and gain clearness by arranging the facts briefly in tabular form, as shown below:

	First train	*Second train*
d	x	$x+6\frac{1}{2}$
r	30	45
t	$\frac{x}{30}$	$\frac{x+6\frac{1}{2}}{45}$

Thus he arrives at the tabular form as a short cut for writing long verbal statements. The making of the form is not to be regarded as a step in the solution of the problem in the sense in which the equation is derived as a step in the solution. Pupils are often under the impression that they have accomplished something when they have drawn the tabular form without any of the data or any further steps of the solution. They have no idea as to what they are to do with the table. They draw it mechanically without understanding or appre-

ciation. Generally this is the result whenever teachers introduce the table before the pupil understands what it means or sees the need for it, or if they force him to use the formal arrangement when he feels quite satisfied that he can do it better by the long and tedious method. In such cases the device does more harm than good. However, if the pupil begins to use it when he understands its meaning it will be of real help to him.

In the foregoing illustrations it has been attempted to show that the values of algebra may be defeated if too much emphasis is placed on the purely mechanical phases and not enough on logical thinking. In the attempt to save time teachers are apt to resort to processes that are purely mechanical. The pupils learn to use them in place of the fundamental processes, and the real meaning is soon forgotten. It is recommended that pupils be taught to add, subtract, multiply, and divide rather than to cancel, remove, clear of fractions, and transpose. The latter terms do not indicate algebraic operations, but merely state the ends to be attained. If used too extensively they will develop in the pupil's mind the conception that algebra consists largely of juggling symbols.

NEWS NOTE

Professor Wieleitner Dead

The announcement has been received of the death of Professor Doctor Heinrich Wieleitner, Oberstudiendirektor am neuen Realgymnasium und Honorarprofessor an der Universitat, München, on December 27, 1931. Professor Wieleitner was well known to all students in the history of mathematics. He was a serious scholar and a clear and accurate writer. His loss will be felt in this country as well as in Munich, where he worked for so many years.

What Modern School Administration Expects of High School Teachers of Mathematics*

By Dr. Lambert L. Jackson, *First Assistant Superintendent of Schools, Newark, New Jersey, Public Schools*

Accepting the modern dictum that all school teachers, high or elementary, are teachers of children rather than teachers of subjects, there is the added responsibility of teaching children something. Hence, there exists some 25 per cent of the total high school staff whose efforts are devoted to assisting pupils to a degree of mastery of elementary mathematics. To be able to perform this function the teacher of high school mathematics must make an appropriate preparation.

Modern school administrative standards in New Jersey require the teacher to complete a four-year college course in which mathematical subjects are pursued for at least two years. In other words, no teacher of high school mathematics is officially prepared for his work who has not studied mathematics for two years in advance of the *courses which he attempts to teach.*

In addition to this subject matter standard, there is the further requirement of professional training. Courses in the theory and practice of education are prescribed so that the teacher may know the accepted ways of putting pupils into possession of mathematical knowledge. This knowledge in theory and practice is the basis for the predicated professional preparation. Then, there is the further requirement that the prospective teacher, before receiving a teacher's certificate, must have a unit of supervised practice teaching which serves as a brief apprenticeship preliminary to certification.

One frequently hears teachers of high school mathematics ask, "What have methods of teaching to do with mathematics?" Possibly this question is partially answered by reference to the following characteristics of efficient teaching compiled by Paul S. Lomax of New York University:

1. Method should be adapted to the pupil's learning process. There should be diversity of methods of teaching suited to diversity of methods of learning. In the final analysis, no two pupils learn in exactly the same way; consequently, they

* Address delivered at the 43rd meeting of the Mathematics Teachers of New Jersey at Atlantic City on November 28, 1931.

cannot be most successfully taught in the same way. Here is a constantly besetting difficulty which a teacher of a group of pupils cannot escape.

2. Method should lead pupils to become self-directing and self-evaluating in their education.

3. Method should stimulate the interest and command the thoughtful attention of pupils.

4. Method should provide for relating the new things to be learned to pertinent and familiar experiences of the pupils. Pretesting should be used.

5. Method should provide for pertinent illustrations by which to explain basic considerations of the course.

6. Method should provide a variety of uses of the new things to be learned in order to develop a pupil power of adaptation. A new thing is well learned only when a pupil can by himself readily apply the new things learned to a different or novel situation.

7. Method should provide for repeated stress on essentials, preferably by calling for repeated application of things learned to new and different situations. A real test is to be found in a new problem.

8. Method should be suited to the nature of the subject matter to be learned—whether it is primarily something to be learned as information, something to be drilled upon as a skill, or something to be practiced as a personal quality or trait.

9. Method should develop in a pupil a feeling of responsibility for successful learning to the class as a group as well as to the teacher. Successful group learning calls for team work.

10. Method should be suited to the teacher's personal qualities and ways of thinking, feeling, and acting. The value of any method is not in itself but *in the teacher* who demonstrates the value.

11. Method should cause the teacher to anticipate the questions of pupils and to prepare carefully thought out answers.

12. Method should be suited to the emotions of pupils as well as to their mental processes. Attitudes are much more powerful and consequential than mere ideas.

13. Method should provide for a general summary of the main essentials of the lesson.

14. Method should provide for careful guidance of pupils in the doing of their assignments. How to do is essential to what to do. And *why undertake the task* is essential to both the other considerations.

That teacher who will think out unaided and apply these criteria of good teaching is conceded to be exceptional. Instruction in method and training in practise are essential to the preparation of teachers of average ability.

This special academic and professional preparation might be interpreted to signify that teachers of mathematics must be truly specialists ("those who know more and more about less and less"). A degree of specialization is obviously intended, but the four-year liberal arts college course presupposes an added culture and breadth of understanding sufficient to insure a sympathy for teachers in other departments. Without this broadening influence, teachers in

any particular field may tend to become clannish and uncooperative.

But, modern school administration is not satisfied with preparation for instruction nor with successful service in teaching. School systems now demand other services, such as curriculum construction, pupil guidance, and contributions to extra-curricular activities.

One might suppose a subject like elementary mathematics to be standardized. It is old enough to be grown up. Further thought, however, reveals need of curriculum adjustments in high school mathematics as well as in other subjects. These adjustments are concerned with "differentiated courses," "unified mathematics," "X, Y, Z levels," and other considerations which have grown out of attempts to meet modern needs. The belief that a rigorous course in algebra is equally suitable as basic work in the academic, commercial, and the technical curriculums no longer holds. School knowledges are specific and are valuable chiefly for their usability. Hence the teacher must know not only "Mathematics," but a wide variety of mathematics so as to assist in formulating a diversified modern curriculum. Then, paralleling this diversification, but not opposed to it, is the need for associating related processes. The different mathematical courses have been treated as separate compartments, as if algebra, geometry, trigonometry, and calculus had no interrelationships. It is true that these subjects present a series of advancing logical steps, but they are not mutually exclusive. Complete unification may not be possible, but related materials exist in algebra, geometry, and trigonometry, or in algebra, geometry, and calculus which are on the same level of difficulty and utility. To make instruction effective some of these correlations should be utilized. So far we have spoken of the logical divisions of the curriculum, but there is also the psychological aspect. When a given course is indicated on the basis of subject matter there remains the task of adapting it to different pupil capacities: with the passing of the high school as a selective institution, there has resulted so great a spread in the mental abilities of high school pupils as to require more than one level of teaching material in any given course. These are some of the considerations which make it necessary for the teacher in a modern school to contribute to the organization of the curriculum as well as to its application in the classroom.

So much for teaching materials, but this is not all, the teachers'

services must be extended to leading or directing the pupils through this maze of learning not only in the field of mathematics, but in related fields, and when this is undertaken, the pupil's present and future ambitions and welfare have to be consulted, leading unavoidably to pupil guidance in the broad sense. In other words, the teacher of mathematics like other teachers in the high school must participate in the task of counseling pupils to the end that they may profit up to their capacities and efforts by the present unlimited opportunities in Public Secondary Education.

Passing from the process of teaching and the process of interpreting, there remains the process of training. We used to speak of the disciplinary value of subjects; now we speak of the disciplinary value of method and the necessity of experience. There is unquestionably as much need as ever for developing in individuals such habits as concentration, industry, and perseverance, but this is no longer entrusted to one set of activities such as the processes of abstract mathematics. It is extended even beyond the curriculum domain into the field of social experience. If education or schooling is to result in right development the school must provide a social régime and opportunities for pupils to participate in such social activities. Whether the pupils gain as much from curricular as from extra-curricular activities is not the issue. The related activities do provide an additional opportunity for civic and social participation which is invaluable in the training of youth. So fully is this recognized by secondary and collegiate institutions that teachers everywhere are expected to make contributions to one or more phases of such school work.

We have now outlined what is expected by modern organization of the teacher within the school; this might be referred to as intramural duty, for there is beyond this the extra-mural or community responsibility. This does not imply that the school system in which a teacher works is entitled to dictate the political, the religious, or social connections which the teacher shall make, but it does imply that the teacher in order to fill satisfactorily his position shall take his place in the community as a cooperative individual and assume his share of civic responsibility. This is no more a requirement of teachers of mathematics than of other teachers, but it is quite as much so. There is a tendency among high-school teachers, particularly in the academic group, to become separatists. They feel that they are making their contribution to society by passing on to the next generation that peculiar culture which they have chosen

to teach. They magnify the social importance of this or that particular subject, and withdraw from activities not strictly in line with the school régime. It is only one step from this position to that of the cult in which ideas, procedures and ideals are static, where tradition is controlling and progress is paralyzed. Even if the isolation does not take this extreme form, it leads to a "better than thou" attitude, which may be the source of the prevalent opinion that high school teachers appear to rate themselves superior to other classes. But granted that they are superior, modern school administration demands that they shall not "flock alone." *The task of educating or schooling all of the children of all of the people is the job of all of the teachers.* Furthermore, such education is a continuous and unified process to be understood and applied by all the teachers in common agreement as to aims, procedures, and outcomes. If there is any distinction between an elementary-school teacher and a high-school teacher, each must know the nature of the other's province so that the pupils' progress may be assured through all the grades on the same terms. If pupils are to know nothing, do nothing and be nothing at one stage, they should know nothing, do nothing and be nothing at all stages. On the other hand, if they are to know something, do something and be something at one stage, they should know something, do something and be something at every stage.

Getting back to the larger community, including the social institutions allied with the schools, many teachers feel that they should be excused from participating in many activities incumbent upon other citizens; but this position is hard to defend. There are two reasons for urging teachers to lead normal civic lives, namely, for the beneficial effect upon themselves and for the advantages accruing to their pupils. The teacher who participates in local organizations and civic movements diversifies his interests, multiplies his satisfactions and stimulates his own growth. He becomes an example to youth and an interpreter of current life, which after all is the most significant educative function a teacher can perform.

Modern educational administration, therefore, reasonably expects of teachers of mathematics preparation in mathematics both academic and professional; it anticipates ability to contribute to curriculum construction, to pupil guidance and to related school activities; and finally, it presupposes that teachers will lead normal social and civic lives in the interests of themselves and their pupils.

Problem Analysis in High School Physics

By KARL A. ZELLER
Central High School, Columbus, Ohio

ONE WONDERS if problem analysis is a lost art, whether teachers perceive its value, recognize its necessity in the development of logical reasoning.

A problem from a recent text is taken to illustrate my point. How long must a force of 100 dynes act on a mass of 20 grams to impart to it a velocity of 40 centimeters per second?

If we solve this problem as stated and simply find, by substitution of values in a selected formula, the required time, then the problem has but little value, and for the teacher who marks the problem right or wrong, depending upon the student's *answer,* it is sufficient.

If, however, in previous classroom demonstration, the group have been shown by proper experiments that when mass and *unbalanced* force applied are constant $v \propto t$; when mass and the time the force acts are constant, then $v \propto F$; and when the force and the time it acts are constant, then $v \propto \frac{1}{m}$; and from these relations derived for the class the formula $v = \frac{Ft}{m}$ and $Ft = mv$, then and only then can the class be expected to visualize and *master* the various functions and their relations, one to another. Otherwise the problem's solution becomes a matter of finding, often by trial and error, what formula to use, what substitution of known values to make. And even then a large number will not be able to solve the equation. Whether it be because the equation does not contain the letter "*x*," one cannot say.

If more time were spent in teaching the fundamentals in algebra, with emphasis on directed numbers, the equation, formula and the graph with its interpretation, perhaps our students would recognize them as valuable allies, when met outside the mathematics classroom. And now if the student sees these relations among the mass, the force acting, the time the force acts and the velocity acquired, he is ready to intelligently attack the problem. Seems to me *some* of our students

should be able to propose and think through the following content or a similar one.

First. What formulas have we derived or developed, in which these given functions are related.

$D =$ average velocity x time

$F = ma$; $v = at$

$$\text{Average velocity} = \frac{0 + at}{2}$$

$Ft = mv$

$Dis = \frac{1}{2}\ at^2$

$$F = \frac{mv - mv_1}{t} = m\,\frac{(v - v_1)}{t} = ma, \text{ from Newton's second Law of Motion.}$$

In all these formulas F must be expressed in dynes, mass in grams, velocity in centimeters per second, distance in centimeters, time in seconds.

Second. In addition to what the problem *asks:*

(a) Why must the force of 100 dynes be an *unbalanced* force?

(b) What distance will the mass move in the time found in the original problem. How far will it travel in one-half this time?

(c) From your answers to (b), what relation do you find to exist between the distances the mass moves and the times the force acts in each case?

(NOTE: the values show a direct relation between the distance and the time squared.)

(d) What velocity will the mass of 20 grams gain each second?

(e) How much will the momentum of the mass change from the end of the third to the end of the seventh second?

(f) Prove that you can apply the formula $F = m\,\frac{(v - v_1)}{t}$ in order to measure the force.

(g) If the force of 100 dynes was removed at the end of eight seconds, how far and with what kind of motion would the mass move, during the next second?

(h) If the mass struck a body in its path, at the end of the eighth second, what would determine how much damage would be done?

Third. What fundamental principles previously studied, appear in this problem?

(NOTE: Newton's three Laws of Motion is one example. A good place to have them restated.)

What, all that time on one problem?—Absolutely!—and more. When you are ready for a problem that will show the amount of work done by a force acting through a distance and the kinetic energy possessed, by a moving body, come back to this same problem, and let the student prove that here Force $\times$ distance $= \frac{1}{2}mv^2$.

You will find this one problem worth more than a dozen. Also that it is "a review" of previous learned situations and experiences in a *new* way.

In this way I believe we can build up a desire for logical thinking, a love for the beauty of reasoning, and a perception of the value and power of functional relations expressed in formula.

Of course students must *be taught* this method of problem analysis, but they love a challenge, and you will find that they will rise to meet this one with a joy and satisfaction that comes from a job well done.

International Congress of Mathematicians at Zurich, Switzerland, September 4-12, 1932

An International Mathematicians Congress will be held in the Federal Institute of Technology at Zurich from September 4-12 next. The scientific part of the program will consist of a large number of general lectures and section meetings. The lectures are intended to provide a complete survey of the present standing of mathematics. Eminent lecturers have, of course, been secured.

The section meetings are intended for briefer discussions of the results of new investigations. The following sections are foreseen: 1. algebra and calculus; 2. analysis; 3. geometry; 4. insurance mathematics; 5. astronomy; 6. mechanics and mathematical physics; 7. mathematical-technical science; 8. philosophy; 9. history; 10. pedagogy.

In addition to these scientific meetings, social gatherings and excursions to different points of interest in Switzerland, are being planned, so that the mathematicians' sojourn at Zurich will be made pleasant and interesting in every respect. Mathematicians from all over the world are cordially invited to participate in this event.

Those from the United States and Canada expecting to attend this Congress are requested to write to Professor F. Conseth, Ecole Polytechnique Fédérale, Zurich, Switzerland, stating their desire to be present and the number of members of their individual families who will be present also.

How the High School Serves the Individual Student*

WHAT ARE our high schools doing for the boy or girl as individuals? Are the exceptional abilities or disabilities of John and Mary receiving more attention today as the result of a modified high school curricula, or does class teaching instead of individual teaching still predominate?

American high schools are varying their programs more than ever before in an endeavor to satisfy in an effective way the peculiar needs of every individual student, according to an announcement made recently by the United States Office of Education.

If Mary is slow to learn she receives special coaching, sometimes in an "opportunity room." If exceptionally above average in intelligence, she is enabled to "skip" a grade or half grade by the provision of special instruction. She may even be placed in an "opportunity room for gifted pupils."

John's failure is investigated, and an endeavor to get him on the right track is made in another special class. Assignments may be regulated according to his ability, or a more homogeneous grouping might be arranged to get him more interested in English or history.

A New Practice in the United States

Twenty-eight different methods by which high schools of the United States recognize and adapt their programs to special needs of individual students were reported to the U. S. Office of Education in reply to 23,000 questionnaires sent to American high school principals in connection with the Office of Education's National Survey of Secondary Education. Preliminary findings were submitted to School Life, official organ of the Office, by Dr. Roy O. Billett, school administration specialist of the National Survey.

Arrangement of the high school program for the welfare of the individual student is very new in the United States, according to Dr. Billett and, with the exception of variation in number of sub-

* The material contained herein from the Department of the Interior ought to be of interest to our readers.—EDITOR.

jects a pupil may carry, and special coaching of slow pupils, few schools consider any provision to be meeting with unusual success. Many schools fail to report any provisions for individual difference in use, he says.

Many Methods

Twenty per cent or more of the larger schools (enrollment over 1,000) use 18 of the 28 provisions originally tabulated on the Office of Education questionnaire. Small schools were found to be less able than the larger ones to adapt their programs to the differing needs of students.

The student is aided in some institutions by advisory programs, psychological studies, adjustment, remedial or restoration classes or rooms. Other differences may be met by out-of-door projects, vocational or educational guidance through exploratory courses, scientific study of problem cases, or various arrangements of study assignment.

Regular class-room procedure has been modified in many instances to meet special abilities or handicaps of John or Mary by the provision of individualized instruction, ability groupings, adjustment classes or rooms, contract, laboratory, or various other plans of instruction. Sometimes credit is allowed for projects or studies carried on outside of school hours by students.

In Large and Small Schools

"Both large and small schools use problem method, project curriculum, out-of-school projects, special coaching of slow pupils, individualized instruction, and laboratory plan of instruction with about equal success," Dr. Billett points out, although he stresses the fact that advisory programs, educational and vocational guidance through exploratory courses, opportunity rooms for slow pupils, scientific study of problem cases, psychological studies, and ability grouping are used in large schools with far greater frequency and reported success than in small schools.

Individualized instruction seems equally applicable in schools of any size, special coaching of slow pupils is practised in more than half of the schools of any size, and successful use of scientific study of problem cases increases as enrollment increases in the high school.

Book Propositions in Teaching Geometry

By AARON HORN
DeWitt Clinton High School, New York City

THE SPECIFIC objectives to be attained in the teaching of geometry, as typically expressed in the New York State syllabus, are of two kinds: (1) the ability to solve "originals" and (2) the ability to reproduce any of a set of "book propositions."

In order to solve "originals" the pupil must be able to employ a series of principles, called propositions, in the deductive discovery of numerical facts, of methods in construction, and of other geometric principles or theorems. In testing the pupil, the original problems set are assumed to be unfamiliar to him. In so far as this assumption is correct, the pupil in passing his examination demonstrates his ability to think geometrically. It is this power of thinking that we aim to achieve and that we hope may have some carry-over effect into the thinking life of the student and future adult.

The reproduction of book proposition proofs is to be contrasted with the solution of originals. While these proofs are supposed to have been originally thought out by the pupil under his teacher's guidance, they involve relatively little creative thought at the final testing stage. On the examination the pupil cannot use his present fund of knowledge to discover, or even rediscover, the proof of a book proposition. The nature of geometry absolutely prohibits this. The proof must be memorized, at least in skeleton, and must be reproduced on the examination essentially as in the book.

This is because the proof of a proposition is not merely a piece of deductive logic, as is that of an original, but is in addition part of a complete system of deductive thought. There should be impressed upon the mind of the pupil the difference between a logical sequence of deduced propositions and circuitous reasoning. He must be made to realize that the reasons on which he bases his original proofs must be built on logical foundations, that a logically deduced fact is unacceptable if the premises are not themselves logically derived. Toward this end we teach the book propositions. And toward this end, it is the contention of the present article that the memorization of individual proposition proofs is of no value.

The proof of a book proposition has no more logical meaning than that of an original problem unless, and only when, this proposition is thought of in connection with the entire system of thought we call geometry. An individual who has memorized the proofs of fifty or sixty book propositions is culturally hardly more advanced than one who has memorized a conglomeration of nonsense syllables unless he is equipped, without any external aid, to fit together these memorized proofs into a complete system.

A person can understand the geometric system without being able to reproduce either a sequential list of propositions or their proofs. He can be tested for his comprehension of this system without being required to reproduce such lists or proofs. At the same time, we can consider it a valuable achievement for him to be able to do so. But, the ability to reproduce the proofs of single propositions, unaccompanied by the capacity to organize them into a complete system, indicates nothing and means nothing.

A pupil is asked on his regent's examination to prove a certain proposition which in his book is, let us say, number 30. He proves it by means of a succeeding proposition, perhaps number 32. The examiner marks him zero. He has not memorized the proof as in the book—he has treated the proposition as an original. On reconsideration the examiner may find that Proposition 32 does not depend on Proposition 30. The student's proof is logically well founded. He is certified as one who knows his geometry—is passed. The teacher may have had to consult his book to verify the correctness of this proof. The pupil discovered it with book closed. Did this pupil understand geometry more fully than the examiner or was his discovery a lucky accident in a stupid teaching and testing situation?

It may be that the syllabus makers think that this boy knew what he was doing. Their assumption may be that the pupil who has memorized fifty proofs and can reproduce sixty-five per cent of them correctly, has really memorized the entire system and can reproduce it if called upon to do so. If that is so, there can be no argument with their objectives, but there can be with their failure to test even an understanding of geometric system. However, the belief, if it at all exists, that these aims are accomplished is, in the opinion of the writer, nothing more than a delusion. Our passing pupils not only reproduce correctly but in all probability understand their required proofs. The best of them probably have a general apprecia-

tion of the significance of proposition sequence. But our guess is that only an insignificant percentage of them could, if put to it, reproduce from memory all proofs in correct logical sequence. If this guess is correct, what means the ability of our passing pupil to reproduce the proofs of isolated propositions?

This article is not at odds with the teaching of demonstrative geometry as a system of thought. It does not argue for the abandonment of proposition study. It does oppose the memorization and reproduction of single, detached proposition proofs. Should we discard this meaningless practice, we could spend more time in trying to give the pupil some notion of what geometry is and in testing his understanding of the relationships among the propositions rather than his memory for the specific proofs of individual ones.

Registration at the National Council Meeting

As the 1932 Annual Meeting of The National Council of Teachers of Mathematics at Washington, D.C., approaches, the secretary is very anxious that the members, guests and visitors register properly. Four different colored registration cards are used to classify our registration. There is behind the color scheme a bit of philosophy which will aid you in determining your classification. The four colors used are green, white, red and blue.

1. *Visitors and guests*, that is *non-members* register on green cards.

2. *Members* who are attending their first annual meeting register on white cards.

3. *Members* who have attended more than one meeting but less than half of the annual meetings register on a red card, indicating that their souls have been kindled and they have caught the fire of the spirit of The National Council.

4. *Members* who have attended *half or more* of the annual meetings register on a blue card, indicating thereby their "true blue" loyalty to the organization. A member does not continue in the blue class unless he maintains his record of attending at least half of the annual meetings. If he does not measure up he slips back into the red class.

The following list indicates when and where our annual meetings have been held:

1920—Cleveland	1923—Cleveland	1926—Washington	1929—Cleveland
1921—Atlantic City	1924—Chicago	1927—Dallas	1930—Atlantic City
1922—Chicago	1925—Cincinnati	1928—Boston	1931—Detroit
		1932—Washington	

Before you go to Washington will you decide which of the annual meetings you have attended and thus facilitate registration by asking for the proper colored registration card. Persons attending any sessions of The Council should register so that the secretary will have his records complete.

A Technique for Introducing General Number and Algebraic Representation of Operation

By HENRY BRECHBILL

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I

THE MAJORITY of algebra teachers and textbooks open their beginning courses in this subject with the treatment of formulas. While the wisdom of introducing the course with so abstract a topic is open to question, it is not the purpose of this article to contend for a departure from the usual practice, but rather to point out certain pupil difficulties and to suggest ways of overcoming them.

The difficulty for the beginner in extending his concepts of particular numbers and their relationships to general numbers and their relationships need not be stressed to experienced teachers of algebra. This is the burden of this first unit of study.

To check on his impression that pupils can solve problems with much greater facility than they can *state how to solve* analogous problems from which numerical values have been omitted the writer tested 101 pupils who had just completed the elementary school. Two sets of six verbal problems each were given. In one set numbers were given and numerical answers asked for; in the other set a problem situation was described and the method of solution was asked for, but no numbers were named. For example, the numerical problem involving multiplication and addition read as follows: "What will 3 bananas and 5 apples cost if bananas are worth 2c each and apples, 4c each?" The analogous problem without numbers was, "How would you find the cost of some tablets and pencils if you knew these things: the number of tablets, the number of pencils, the cost of one tablet, the cost of one pencil?" The problems in each set were arranged in random order.

Table A shows the operations involved in each set and the percentage of incorrect answers obtained from the elementary group and the first year high school group computed to the nearest whole number.

TABLE A. PERCENTAGE OF ERRORS IN SOLVING SIMPLE PROBLEMS AND IN STATING HOW TO SOLVE ANALOGOUS NON-NUMERICAL PROBLEMS

	High School Group		Elementary Group		Both Groups Combined	
	Numerical	Non-Numerical	Numerical	Non-Numerical	Numerical	Non-Numerical
Addition	0	1	1	2	1	2
Subtraction	13	14	13	21	13	18
Multiplication	0	5	3	35	2	21
Division	4	9	11	19	8	14
Multiplication and Addition	3	10	14	37	9	25
Addition, subtraction and division	4	21	17	53	11	40

In each group, a much larger percentage of failures is found in the non-numerical than in the numerical problems. A year in the study of algebra has reduced the number of errors in both types of problems. Since algebraic representation of quantitative relationships requires the ability to comprehend those relationships even though no numbers are specified, it is clear that this difficulty of pupils needs to be clearly recognized and intelligently provided for.

II

The formula involves not only the abstract concept of general number and its relations, but the practical art of representing these relationships. The algebra teacher who assumes that skill in the use of the conventional methods of representing the fundamental operations or even a knowledge of such methods is assured by the previous study of arithmetic is foredoomed to disappointment. Since the process of addition in arithmetic is usually accomplished by placing the numbers to be added in vertical columns and the sum below them all and separated from them by a line, this form might be expected to express addition to the arithmetic pupil. Division is suggested to him by such a form as $5\overline{)75}$ or, perhaps, $75 \div 5 = 15$, but only in rare cases by $\frac{75}{5} = 15$. Multiplication is indicated to him by the ordinary

position for the operation, $\begin{array}{r} 15 \\ \underline{5} \\ 75 \end{array}$, or by the use of the symbol $\times$, as in $15 \times 5 = 75$. He is seldom acquainted with the use of the dot to indicate the process $(15 \cdot 5 = 75)$, and of course, cannot have had experience with mere juxtaposition, (xy) as indicating a product.

The same groups of pupils referred to above were also given the following direction: "18 added to 14 makes 32. Set down this fact in all the different ways of which you know for *showing addition* and place a check mark at the way you think best." Similar instructions were given for "showing" the other fundamental operations. The results are stated in the table on page 82.

The significant thing which these figures reveal is the unfamiliarity of pupils with the standard methods of representing the fundamental relationships. Of the elementary group 74 per cent knew how to represent addition in the form, $14 + 18 = 32$, but only 3 per cent thought it the best way. $17 - 8 = 9$ as a way of expressing subtraction occurred to 66 per cent but was rated best by only 2 per cent. The fractional form for division and juxtaposition for multiplication are extremely unfamiliar. Even a year of algebra as taught to these children did not establish these fundamental conventions of mathematics.

In as much as sound pedagogy consists in taking one's departure from that which the learner already knows, presenting the new in discrete portions and thoroughly integrating it with the old, the following procedure is suggested, a procedure which has been used and found effective. First, a simple numerical problem is assigned such as "Find the number of pupils in a class consisting of 11 girls and 15 boys." After the answer 26 is announced, the class is directed to indicate the process using the proper form, $15 + 11 = 26$. Similar exercises are used until the solution of any problem using a single operation can be correctly indicated. Of course the fractional form for division is insisted upon and the dot, $\cdot$, is used for multiplication rather than the cross mark, $\times$. The latter measure is taken as an intermediary step to the elimination of the sign entirely when literal notation is introduced.

When satisfactory proficiency has been achieved in indicating single operations, problems involving combinations of operations are attacked, such as these:

(a) A contractor employed 7 carpenters and 3 masons, paying each man 8 dollars daily. What is his payroll in a 5 day week?

TABLE B. PERCENTAGE OF PUPILS KNOWING AND PREFERRING VARIOUS METHODS OF REPRESENTING THE FUNDAMENTAL OPERATIONS

Forms of representing	High School Group (81)		Elementary School Group (101)	
	Known	Preferred	Known	Preferred
Addition $14+18=32$	98	20	74	3
$\begin{array}{r} 14 \\ +18 \\ \hline 32 \end{array}$	79	60	60	53
$\begin{array}{r} 14 \\ 18 \\ \hline 32 \end{array}$	26	20	39	31
Erroneous forms	1	1	3	1
Subtraction $17-8=9$	91	18	66	2
$\begin{array}{r} 17 \\ -8 \\ \hline 9 \end{array}$	91	59	79	66
$\begin{array}{r} 17 \\ 8 \\ \hline 9 \end{array}$	9	8	29	22
Erroneous forms	11	0	12	0
Multiplication $14\times16=224$	96	15	76	2
$(14)(16)=224$	21	0	0	
$14\cdot16=224$	51	1	0	
$\begin{array}{r} 14 \\ \times16 \\ \hline 224 \end{array}$	79	67	68	58
$\begin{array}{r} 14 \\ 16 \\ \hline 224 \end{array}$	25	18	32	24
Erroneous forms	1	0	2	
Division $624:24=26$	0	0	0	
$624\div24=26$	73	11	40	2
$\begin{array}{r} 624 \\ \hline =26 \\ 24 \end{array}$	31	3	21	3
$\begin{array}{r} 26 \\ \hline 24/624 \end{array}$	98	79	93	71
Erroneous forms	25	0	29	1

(b) John wishes to divide 150 peanuts equally among his 2 brothers and 3 sisters. How many will he give to each?

These problems result in such forms as, (a) $5 \cdot 8\ (7 + 3) = 400$ or $5 \cdot 8 \cdot 7 + 5 \cdot 8 \cdot 3 = 400$ and (b) $\frac{150}{2 + 3} = 30$.

After problems of a fair degree of complexity can be stated in this manner, preparations are made to launch the general number idea. Several easy problems, such as calculating the area of a rectangle are given, differing only in the numerical values assigned. Attention is called to the similarity in form of the resulting statements. The pupils are led to declare that any problem of that type would be stated in the same form. The teacher then suggests that that general truth could be shown by using characters which would mean any number. The pupils usually propose the use of letters. Thus, formula after formula is developed, always from several particular problems whose numerical solution has been indicated in proper algebraic form. Since the latter has been introduced to meet a felt need for something to represent number it is very easy to reverse the process and substitute numbers for the letters.

◆ NEWS NOTE ◆

Twelfth Annual Ohio State Educational Conference

Walter Lippman, of the New York *Herald-Tribune,* formerly Associate Editor of the *New Republic,* and Charles H. Judd, Head of the Department of Education and Director of the School of Education of the University of Chicago, will address general sessions of the Twelfth Annual Ohio State Educational Conference to be held in Columbus, April 7, 8, 9, 1932. An attendance of 6,000 is anticipated. "Education as Social Investment" will be the keynote of the two general and the thirty-six sectional meetings.

W. S. Schlauch of New York University will speak to the High School Principals group and to the Mathematics Section.

Club Topic: Symmetry

By NORMAN ANNING

University of Michigan, Ann Arbor, Michigan

THE STUDENT who is preparing to discuss such a topic in mathematics club will find plenty of suggestions in F. C. Boon's *Companion to Elementary School Mathematics*, Longmans, Green and Co., 1924. Mention is made, as it is in any good textbook, of the bilateral symmetry of the coefficients of the expansion of $(a+b)^n$ where n is any positive integer. It is the purpose of this note to suggest that this fact is a member of a hierarchy of facts which an interested student would find easy to explore and to explain. The restriction of the exponent n to positive integers remains throughout.

The coefficients of the expansion of $(a+b+c)^n$ can be arranged in such a way as to exhibit a symmetry akin to that possessed by the equilateral triangle. An example will be given at the end.

For the expansion of $(a+b+c+d)^n$ the possible symmetry is that of the regular tetrahedron. The student should make a model.

For $(a+b+c+d+e)^n$ the possible symmetry is that of the regular pentahedroid. No model of this four-space figure is possible; it has five vertices which are joined by ten line-segments to form ten equilateral triangles which are the faces of five regular tetrahedra. But models of parallel "plane" sections can be made and studied.

Example: The complete expansion of $(a+b+c)^6$ can be obtained by inserting plus signs and proper powers of a, b, and c in the sketch given below. When we know one-sixth (one-sixth is the reciprocal of factorial three) of the diagram the remainder can be filled in by symmetry.

$$
\begin{array}{ccccccccccccc}
1a^6 & & 6a^5b & & 15a^4b^2 & & 20a^3b^3 & & 15 & & 6 & & 1 \\
& 6 & & 30a^4bc & & 60a^3b^2c & & 60 & & 30 & & 6 & \\
& & 15 & & 60 & & 90a^2b^2c^2 & & 60 & & 15 & & \\
& & & 20 & & 60 & & 60 & & 20 & & & \\
& & & & 15 & & 30 & & 15 & & & & \\
& & & & & 6 & & 6 & & & & & \\
& & & & & & 1 & & & & & & \\
& & & & & & 84 & & & & & &
\end{array}
$$

Mathematics Journals

By WALLACE SMITH

BECAUSE of the interest voiced by the teachers of mathematics at the recent meeting of the West Virginia Council of Mathematics during the S.E.A., I am induced to comment on the value of two bulletins published by and for mathematics teachers.

THE MATHEMATICS TEACHER is the official organ of the National Council of Teachers of Mathematics, 525 West 120th Street, New York City. This journal is devoted to problems of teaching in both high school and college. It costs only two dollars per year. These are some of the subjects treated in the November number:

High School Mathematics Teachers in the U.S.
An Experiment in Classification of Students in Mathematics.
Solid Geometry in the High School.
Maintenance Drills in the Junior High School.
A Seventh Grade Mathematics Class Bank.

There are other topics just as interesting and important as these. They speak for themselves as to value and the need of having access to them.

The second bulletin is *The American Mathematical Monthly,* the official journal of the Mathematical Association of America. The Secretary-Treasurer of the Association is Professor W. D. Cairns, 33 Peters Hall, Oberlin, Ohio. This journal costs four dollars per year to members of the Association. It costs two dollars to enroll as a member. This journal contains some papers that are more advanced than the one previously mentioned. Its department of problems and solutions should be of extreme interest to young teachers. Many of the topics treated are elementary and can be read by those having had only calculus.

If a teacher of mathematics expects to depend on that vocation for his joy and sustenance, then it behooves him to become familiar with these two journals. A ship without a pilot and rudder is sure to flounder among the waves and crash on the rocks. If a teacher of mathematics is without access to current literature containing the ideas and work of our best teachers and mathematicians, then he has neither pilot nor rudder. THE MATHEMATICS TEACHER will inspire the teacher to correct some faults and improve some weak points.

The *Monthly* will lead him to consider more advanced problems, which every teacher must do or die professionally. He cannot remain stationary. He must grow or retard. If he does not care to grow, then he will be forced out of the profession by those coming on who do want to grow and improve.

Mathematics is in greater demand today than ever before in the history of the world. Every modern science is seeking to express its laws and truths in the language of mathematics. The youth must acquire a reasonable familiarity with this language if he hopes to play his part in modern life. It is the duty of every high school teacher of mathematics to prepare to give the best instruction possible. One must inspire to teach, and to inspire he must in turn be inspired. He not only must be inspired but he must also be able to solve many unique situations and demonstrate to his class that he is familiar with his subject. Young teachers may be aided in these undertakings by having recourse to the recorded experiences of others.

These two journals herein mentioned will make this possible. He will make contact with many outstanding teachers and mathematicians by reading some of their pages, which will indeed be an inspiration. There is another item of much interest. Both journals publish brief reviews of new books. These will be useful in choosing new books for adoption. Six dollars per year and a silent conference of thirty minutes daily with the contributors to these journals will work wonders with any teacher of mathematics.

Don't Forget the Meeting

of the National Council of Teachers of Mathematics at the Raleigh Hotel in Washington, D.C., on February 19 and 20, 1932. The complete program of the meeting will be found on pages 47-50 of The Mathematics Teacher for January.

Junior High School Mathematics

Not Yet Arrived But on the Way

By EDITH L. MOSSMAN
Garfield Junior High School, Berkeley, California

WHAT IS A real junior high school? What is it inherent in adolescent boys and girls, that has led psychologists and progressive teachers whether students of formal psychology or not, to urge the change from the 8-4 plan to a 6-3-3, 6-4-2, or 6-4-4? What are the desires, habits, and necessities of the earliest *teens* that should greatly influence both *content* and *method* in all junior high school subjects?

Does the customary seventh and eighth grade arithmetic, followed by ninth grade algebra, each with a few difficult, unpractical, and abstract topics omitted, and a few topics such as personal accounts, budgets, and graphs added, truly constitute a "junior high school mathematics course" just because offered in a junior high school building? Again, have the textbooks on general mathematics or so-called junior high school mathematics solved the problem?

Why not eliminate much more subject matter which is now included? Are we failing to give the boys and girls a "square deal" if we do not present to them many of the present topics which certainly do not appear to be very contagious, and which they do not assimilate at all as far as results indicate? Are there not other topics of much greater life value than some of those now in the course? For those topics retained, are there not *methods* of presentation that, while requiring more time, will give the pupils mental experience of far richer value?

Teachers can and should learn something from school systems with ideas very different from ours such as those modern schools which have no separate course in arithmetic. Such schools teach essential facts in mathematics only as the need arises in connection with various practical projects. What can we learn from certain European schools—and some in the United States—that begin algebra and geometry at the same time, and earlier than we do in most of our public schools? What is done in Kansas City, for example, where they claim to accomplish thoroughly in seven years as much mathematics as we Californians do in eight?

Probably it is not desirable that we should adopt any one of these plans *in toto*. Surely, instead of pushing pupils ahead more rapidly, we shall have less skipping of grades. We shall allow even those of highest I.Q.'s to remain as normal children among others of their age, but with a greatly enriched course. However, it may be true that a study of these plans with their far from tragic results may help to pry us loose from our rock-like conservatism.

The business world criticizes us because we do not teach fundamental points thoroughly. Many thinking men and women in all walks of life are convinced that little value has resulted to them from the courses in mathematics which they were required to take. More than a few intelligent, successful people say they never knew "what it was all about"—that their time in mathematics was largely wasted.

Throughout our schools, from the elementary grades through college, it is often said whether true or not that the majority of those who are discouraged, who have an "inferiority complex," have been driven to that sad state through failure in mathematics. In no other subject is there found as great a number of normal, conscientious boys and girls who, though willing to work hard, are nevertheless unhappy, floundering, uncertain, frightened, unsuccessful.

But enough of these questions! Let us remember, however, that one of our great men said something to this effect: The human mind, especially that of a pedagogue, needs periodically to be turned topsy-turvy, well shaken up, like a feather bed. If these questions have performed that service for us, we are ready to discard a number of cherished schemes, in order to reach for something better. We should be wise enough to avoid unproved, fanatical wastes of time. We should be able to develop something of genuine progress, based both upon the needs of the new age, and upon a direct contact with and an understanding of boys and girls.

Many sincere teachers of earlier days objected strenuously to giving up *"lots* of Latin for *all."* Today many teachers hesitate to throw out half or more of our present course in junior high school mathematics. We have lost sight of the fact recognized in the formation of this unit—that *here* is a period of *exploration,* not of technical training. A "smattering of this and that"—slip-shod, superficial work, of course—must not be tolerated. But these adolescent boys and girls should not be forced to take *all* that is now required. Neither should they be denied some glimpses of geometry and trigonometry.

You and I who deeply love beauty in all phases of nature, in pictures and statuary, in homes, in people of all ages from infancy to later years—but who have little or no talent to express beauty through paint brush or pencil—just suppose that we had been forced through perhaps twelve years of school life to spend an hour or more a day on various technical branches of painting. Try with all our might, under the best of teachers we seldom, if ever, could have made *Honor Roll* grades. Some in our classes would have excelled. The rest of us would not have reached the requirement; we would have been denied entrance into college, and would have developed various unhappy attitudes and habits.

However, instead of this, we were given simple, practical courses in color, perspective, design, art appreciation. All of us, who were interested and willing enough to try, did creditable work and were happy in it. Furthermore, we gained something practical, something of an uplifting, broadening, and thrillingly appreciative effect, that will be of great value to us all our lives.

During the past five years, in my junior high school mathematics classes and clubs, I have been experimenting with various kinds of *appreciative* lessons, some of which deal with the following:

1. Stories of earliest development of number in the consciousness of men.
2. Wonderful early accomplishment in Egypt, Babylonia, and China.
3. Later contributions from other parts of the world.
4. Biographies of mathematicians, past and present.
5. Plays from THE MATHEMATICS TEACHER.
6. Discussion of the topic: "Suppose all knowledge of mathematics should disappear from the world."
7. Work in posters, slides, modelling, and clippings.
8. Encouragement in observing mathematics in nature and in human affairs.

The pupils have been eager and enthusiastic about these lessons, and in nearly all cases showed markedly greater interest and effort thereafter even in the regular topics.

In the near future, when *real* junior high school mathematics arrives, a good part of it, perhaps a third, will be in the line of appreciation.

Mathematics teachers have looked upon the report of the National

Committee almost as a Bible. That report states plainly the moral, ethical, and spiritual values, possible in mathematics courses. These great possibilities, however, are as yet almost untouched in actual practice.

Point out to your classes that frequently what is studied as fact in the textbooks of one generation, is proved to be mistaken by the thinkers of a later period. Then show them a facsimile of the Rhind Papyrus, bringing to their attention that Ahmes, writing a mathematics text about 4000 years ago, presented nothing that can be contradicted today.

It is well to start a discussion of (1) those things we study which are more or less man-made and so subject to change, correction, improvement, and (2) those which are God-made, unchangeable, discovered little by little—in no way dependent upon man or matter—existing as true throughout time, and in both inhabited and uninhabited parts of the universe.

Some of the ninth grade boys and girls thoroughly enjoy the year's work in algebra. But to an appalling number, the greater part of it is a hideous nightmare. All could be much happier and could gain immeasurably more life-time value if, instead of the present quantity of algebra, there was given simple, but *thorough, meaningful,* experience in the several different fields of arithmetic, algebra, and geometry both intuitive and demonstrative.

Little excursions should be taken into trigonometry. A bit of a view—some definite idea of what is done there—might be given into the world of analytics and calculus. Many children who will never go deeply into anything mathematical, are denied the slightest knowledge of these things. That is just as narrow and unfair as it would be to deny you and me any knowledge of the various phases of art, because we can never go deeply into it, can never excel in it. But if in this subject we inquired into the question of *content* only, a point of vital importance would be overlooked, i.e., *method* of presentation. Much time is worse than wasted in unproductive drill—in solving a great many problems of the same type, in a "follow the leader" style, unconnected with active thinking, and with no results beyond the immediate grades recorded.

The following experiment was conducted by a certain school: A list of questions was sent out to the sixth, seventh, and eighth grades of a great many schools. Nearly all of the questions called for the most sim-

ple facts drilled upon over and over in all arithmetic classes. But among these questions were just a few that though mathematical in character, were nevertheless not found in the courses of study of these schools. Some called for memory of facts, others for understanding of procedure in arriving at the correct answer. Thousands of lists were corrected and the results tabulated. They were such as to make one gasp. Outside the school walls, such appalling waste of time in drilling for something never accomplished, would not be tolerated.

And here is an extremely interesting fact that has made me improve my class room procedure since reading this experiment several years ago. Those points that had *not* had any class room drill gave better results than those that had had such drill. Part of the children knew nothing about these outside questions—had never heard of them—so left the space blank. But the many to whom they were not wholly unknown, answered perfectly in nearly all cases.

Why did the same children who succeeded in getting quickly and thoroughly those outside points, fail in other points upon which they had spent much time in school? Why will an intelligent boy or girl allow to go unchallenged an astonishingly absurd answer, although out on the street he would not for a moment be fooled into accepting such nonsense as true? Here are some common illustrations of this: (1) A ton of coal costing $1,750, (2) 0.37 subtracted from 90, giving a result of 53 or 0.53, (3) Interest for a year larger than its principal.

Children outside of the school room learn quickly and thoroughly points as simple as these, and difficult ones too. They are seen "sizing up a situation" with clear judgment, and drawing reasonable conclusions.

Why do we fail to provide for the same kind of mental contact that elsewhere brings good results? A large part of the trouble lies in the fact that so much of the time, so many of the pupils are more or less blindly following a form, not really knowing the why or wherefore, not led to wide-awake, thorough thinking about the situation.

During recent years, my classroom procedure has improved along the line of *fewer* problems solved. These few are done in *such* a *way* that certain lasting value is realized for each pupil. Definitely better results have followed, both in the increased happiness of a larger number of children, and in the amount of clear and accurate thinking.

In this effort it has been found valuable to give credits as follows:

1. The estimate made *before* beginning the solution on paper, to avoid being satisfied with an absurd answer, and to make sure that the steps have been thought through before the pencil has been used.

2. A good, well-proportioned drawing whenever possible.

3. Definite, short, abbreviated labeling of result obtained by each process.

4. Correct answer.

5. Proof.

6. Another solution by a different method if possible.

7. Business-like neatness and plainness of entire paper.

8. Correct English forms: spelling, periods, commas, capitalization in heading of paper and in solution.

More time in reading problems and in telling *what* to do and *why* has proved worth while. Short, frequent, snappy, meaningful drills for accuracy and speed have not been omitted. Long lessons to prepare outside of class have been supplanted by those asking for only from two to five problems to be prepared in the way that provokes the greatest amount of careful thinking.

The theory of mental discipline, as expounded years ago, has been exploded. Nevertheless, psychologists agree upon a vital point in transfer of habit. A most decided awakening and improvement follows a class discussion of these two questions:

1. Which has greater value to the average boy or girl (a) the learning of mathematical facts and processes, or (b) the various good habits—attitudes of mind—that may be formed or strengthened through the mathematics courses?

2. What are some of those good habits and attitudes?

The members of the class will name: neatness; independent thinking; accuracy; alertness to follow directions exactly; honesty; appreciation; straightforward, business-like procedure; determination to correct all mistakes; quick discernment of absurd conclusion; and reverence. And they will not only name these, but such a discussion is followed by a surprising amount of awakening to better effort and more happiness.

In summing up, let us say that a course in real junior high school mathematics will discard that which is needed only in the training of a specialist, and that which is discouraging to the average, normal, intelligent youth.

It will supply well-sharpened tools that shall have reached their

fine point through meaningful drill, omitting all waste of time in unthinking following of form.

It will give simple, practical applications of fundamental points in problems finding background in the pupil's experience, and appealing to his interest.

It will provide valuable, joyful excursions into the simple introduction to algebra, geometry, trigonometry, and perhaps analytics and calculus.

It will definitely develop each pupil along the line of appreciation, of culture. It will employ a method that accomplishes something good morally and spiritually.

It will be conducted under such a plan as shall keep clearly in the consciousness of the young people, the good habits and attitudes of mind which they are having the opportunity to acquire and strengthen.

Every boy and girl will learn that there are no exceptions to the rule that the right result must follow right steps leading thereto. Conscientious effort and correct attack must lead, here as elsewhere, to deserved success and happiness.

Bonaventura Francesco Cavalieri

1598-1647

CAVALIERI, a student of Galileo, a Jesuit priest, and professor of mathematics at Bologna, is known chiefly for his work with indivisibles and for the theorem called by his name. He might also be remembered as one who did much to make Napier's work with logarithms known in Italy. In the picture which forms the frontispiece of this issue of THE MATHEMATICS TEACHER, reference is made to Cavalieri's discovery of the theorem for the area of a spherical triangle. The fact that Albert Girard had discovered the same theorem at about the same time does not imply that either was aware of the work of the other.

Cavalieri's *Geometria indivisibilibus continuorum nova quadam ratione promota* was used as evidence of his fitness for the professorship at Bologna in 1629, but the volume was not published until 1635. In this work, a point is considered as the indivisible of a line, a line of a plane, and a plane of a solid. A moving point generates

a line and so forth. An important instrument in the study of indivisibles was Cavalieri's Theorem:[1]

> Any plane figures constructed between the same parallels, in which [plane figures] any straight lines whatever having been drawn equidistant from the same parallels, the included portions of any straight line are equal, will also be equal to one another; and any solid figures, constructed between the same parallel planes, in which [solid figures] any planes whatever being drawn included within these solids, are equal, the [solid figures] will be equal to one another.

Applications of this theorem lie within the high school field as is witnessed by its use in determining the volume of a sphere.[2]

Indivisibles enabled Cavalieri to solve many of the problems proposed by Kepler and others for while his concept of indivisibles lacked scientific foundation, it served as a crude sort of integral calculus. By it, for example, Cavalieri developed a formula equivalent to finding the value of $\int_0^1 x^m dx$ where m is a positive integer.

The Swiss mathematician Paul Guldin (1577-1643) had rediscovered a theorem which is now called by the name of Pappus (*c.*300). The theorem states that the volume of a solid of revolution is equal to the product of the area of the generating figure and the circumference of the circle traced by the center of gravity of the generating figure. Cavalieri pointed out the weaknesses of Guldin's proof and provided a proof of his own. Guldin retaliated by attacking Cavalieri's work. Accordingly Cavalieri revised and refined his theory printing his new statements in 1647 with the title *Exercitationes geometricae sex.*

[1] *Source Book in Mathematics,* New York, 1929, p. 605. See also a translation by G. W. Evans, *American Mathematical Monthly,* XXIV, 447.

[2] See for example, Beman and Smith *Solid Geometry.* Here the area of a section of a sphere cut by a plane parallel to the base of the circumscribed cylinder is equal to the difference in the areas of the sections cut by this plane from the circumscribed cylinder and from the cone whose base is the base of the cylinder and whose vertex lies at the center of the sphere.

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Arithmetic in Teacher Training

By BEN SUELTZ
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EXCEPT FOR THE fact that arithmetic is a subject for study in all or nearly all of our elementary schools we perhaps would not be spending any of our valuable time in the teacher training institutions in preparing the neophyte teachers for the rather serious task of teaching arithmetic. Isn't it curious that arithmetic is being taught so prominently in our elementary levels of instruction? Have not we all heard reformers say that all the arithmetic needed by the average citizen can be learned in six months? Why then do we spend eleven and six-tenths per cent of the school time in grades one to six in the study of arithmetic?[1] The writer does not wish to enter fully into the arguments of this controversial question but in order to give an historical setting for the material which will be presented later, it seems wise to point out at this time that arithmetic was once considered too vulgar to be included in the elementary school curriculum. A fuller discussion of the beginnings of arithmetic as a school subject would show how our term arithmetic gradually came to include both ancient arithmetic (theoretical work with numbers) and logistic (practical calculation) and how for a long time the schools of non-commercial Europe looked at this study as one to be pursued by only that menial class of people who stooped so low as to gain a living through barter and commerce. Also an historical consideration would show how arithmetic grew in prominence and importance as the industrial revolution began to peacefully transform the point of view of the Western world from which we have taken our heritage of social institutions as well as of our physical beings.

In the settling of New Amsterdam the Dutch who were already a commercially minded people brought with them the study of arithmetic as is indicated by the following statement which gives instructions to the schoolmasters going to the East and West Indies (date 1636).

> He is to instruct the youth—in reading, writing, cyphering, and arithmetic, with all zeal and diligence; he is also to implant the fundamental principles

[1]Carlton H. Mann, *How Schools Use Their Time*, 1928, p. 78, table 16 A.

of the true Christian religion and salvation, by means of catechizing; he is to teach them the customary forms of prayers, and also accustom them to pray; he is to give heed to their manners and bring these as far as possible to modesty and propriety.[2]

Three hundred years ago the Dutch realized not only the usefulness of cyphering and arithmetic but they also realized that the mere knowledge of prayers did not signify the habit or custom of praying.

In the settling of New England the English who came largely for religious reasons considered reading and limited language arts as uppermost in importance because these were directly useful in extending the knowledge of religion. The Massachusetts law of 1647 provided for schools in towns of fifty householders by appointing "one within their town to teach all such children as shall resort to him to write and read." The same law also provided for grammar schools in towns of one hundred or more families and in these schools which were considered of secondary rank the master was qualified as "the master thereof being able to instruct youth, so far as they may be fitted, for the university."[3] As commerce grew in the colonies arithmetic grew more and more popular and in due time the universities began to exert their influence upon elementary education. Whether for evil or for good in 1745 Yale required "common arithmetic" for entrance and in 1760 Princeton required candidates "to understand the principal rules of vulgar arithmetic."[4] In 1807 Harvard required that candidates for admission "be examined by the President, Professors and Tutors. No one shall be admitted unless he be . . . well instructed in the following rules of arithmetic, namely, Notation, simple and compound Addition, Subtraction, Multiplication, and Division together with Reduction and the single Rule of Three."[5]

Early in the nineteenth century we had a significant popularization of the study of arithmetic which was reciprocally met by the equally significant development of American textbooks in arithmetic. Many of the texts which were produced in the early part of the century show clearly that the writers were aware of the social and commercial life about them, but the types of exercises used show also that these

[2] W. H. Kilpatrick, *The Dutch Schools of New Netherland and Colonial New York*, 1912, p. 220.

[3] E. G. Dexter, *History of Education in the United States*, p. 585.

[4] E. E. Brown, *The Making of our Middle Schools*, p. 129.

[5] E. C. Broome, *A Historical and Critical Discussion of College Admission Requirements*, p. 42.

scholar-writers were deeply interested in the old theoretical arithmetic of the Middle Ages. At the middle of the century the problems that were included in the books were often highly fictitious and the real practical application of arithmetic seemed to be in the background. This persisted until the close of the century when reformers began to clamor more loudly for an arithmetic which would teach those processes that are most likely to be encountered in the daily lives of the citizens. Thus it has continued until the present day. In spite of all the new subjects, the reforms, and the manifold scientific investigations in the field of education that have come into our elementary curriculum since the time of the early colonies, we still require children in our elementary schools to spend more than one-tenth of their school time studying arithmetic. Does it seem curious that such is the practice of the twentieth century when in the early seventeenth century arithmetic wasn't even considered a worthy subject for study?

One current point of view maintains that elementary school practice is not at all in harmony with our modern civilization and that curricula are a century behind the times. An opposing group would not only uphold but also extend the present amount of arithmetic, for its value in modern civilization. Perhaps a fair rationalization of present practice may be found in the thought that this is a vulgar commercial age and arithmetic is still the most universal language of commerce.

Without presenting a brief for or against the inclusion of arithmetic in the elementary curriculum and also without considering the nature of this arithmetic, since it actually does exist, it appears quite evident that we must have teachers of arithmetic. (Is there a school where teachers are totally abolished?)

Though not always admitted, nevertheless it is true that those who have the privilege of working in teacher training institutions also have the responsibility for really training teachers. In this article the writer wishes to present some views and points of view concerning courses which aim to make the student in the normal schools and teachers colleges better teachers of arithmetic than they would have been had they not studied such courses. Is it really necessary that prospective elementary teachers study some kind of arithmetic during the process of their training? The answer here may be yes and no, with more yes than no, as shown by the following:

> There is no unanimity as to the best way to train a teacher. There are those who believe that a teacher needs no training at all: that he is born, not made.

. . . There is some evidence in support of this point of view. There are many able teachers who are innocent of professional training. In the main however it seems probable that there exist knowledges and skills that can be taught that serve to reduce the period of apprenticeship of the beginner. Most countries of the world by common practice have agreed in providing some sort of special training for those who teach.[6]

Further, it is interesting to note that Taylor reports that one-sixth of the one hundred eighty-seven schools which are training elementary teachers are giving no course in arithmetic.[7] In the same report we find this statement "the schools in which arithmetic is approaching a vanishing point are nearly all in the part of the country where the name teachers college has recently become popular." As we increase the amount of time required for the elementary teacher's training, shall we increase or shall be diminish the amount of time given to her training to teach arithmetic? At present according to a study by Class[8] we find the four year course giving very little more time to arithmetic than the two year course, and the three year course giving more time than the four year course. Isn't there a real need for such a course when the subject ranks second in amount of time consumed in the elementary school? Perhaps the usefulness of such a course depends somewhat upon the type of course suggested.

Let us assume, as did the greater number of teacher training schools during the past seventy-five years, that some type of course in arithmetic is desirable as a part of the training of an elementary teacher who will later teach this subject to children. Despite the fact that there is relatively close agreement in the United States as to the content of the elementary school course in arithmetic there seems to be no general agreement as to the best type of course for the normal school. Randolph studied one hundred schools for teachers in 1921 and lists the following as his results relative to arithmetic.[9]

Type of Course	*Number*
Review of elementary material	29
Advanced academic study in the subject	12

[6] William F. Russell, *Report of the Dean of Teachers College*, June 30, 1931, p. 18.

[7] E. H. Taylor, *Arithmetic Teachers in the Making*, The Teachers College Bulletin, Charleston, Illinois, 1928, pp. 13-14.

[8] E. C. Class, *Prescription and Election in Elementary-School Teacher-Training Curricula in State Teachers Colleges*, p. 42.

[9] Edgar D. Randolph, *The Professional Treatment of Subject Matter*, p. 102, table 1.

Methods in elementary material 58
Advanced study of the subject matter but including items tending to professionalize it 7

If we should regroup the four types of courses as given by Randolph under the two headings of Material and Method, respectively, we would note that Method is a bit more popular as a type. This is in harmony with the results as reported by Taylor in his study of 187 teacher training institutions.[10] He gives the frequency of courses as Subject matter 46, Methods 95, Subject matter and Methods 112. Professor Upton[11] also calls attention to the fact that the teachers' courses in arithmetic are in the main similar to those offered twenty-five years ago while conditions in the elementary schools have changed very markedly during that time. In general three types of courses are found in our training institutions. These are the Materials course, the Methods course, and the combination Materials-Methods course.

Although it is difficult to learn exactly what is being taught in any school by examining the catalog of the school, yet such an examination of catalogs does show what teacher training schools say they are doing. It is their official statement of what someone (often not the teacher) thinks is being or ought to be done. From a limited sampling of catalog descriptions of the three most common types of arithmetic courses the following generalized descriptions have been made.

The Materials Course in arithmetic is one that aims to teach arithmetic of an elementary nature or it aims to teach advanced arithmetic beyond the level of the elementary school or it aims to do both and often, either consciously or unconsciously it gives very definite ideas of method. These elements of method may be concomitants which develop germainely out of an instructor's presentation and they are learned through the potent social medium of imitation. Nevertheless the main aim in a Materials course is to teach subject matter of arithmetic. Those who hold to this type of course are prone to point out that many of the high school graduates who enter the normal school do so with a sixth grade achievement in arithmetic. There are others who hold to the Materials course for the good old reason that "you can't teach a thing unless you know something about it." It has been the writer's experience in measuring several

[10] E. L. Taylor, *op. cit.*, p. 5.

[11] C. B. Upton, *The Teachers' Course in Arithmetic in Normal Schools*, Teachers College Record, October, 1925.

hundred normal school freshmen in New York State by means of several different "standards of mediocrity" (standard tests) that the average achievement level is somewhere in the Seven A grade. This is very satisfying when it is remembered that in most cases no arithmetic has been studied for four years and that the highest level of achievement ever attained by these people was perhaps about 8 B grade.

The Methods course as usually defined or described aims mainly to teach the students how they may in turn teach arithmetic to elementary school pupils. In this course method is not wholly abstract: it is built out of and around the topics of arithmetic and in this way incidental review of the content of at least some of the topics is provided.

The writer knows of one school where the instructor has each student keep a notebook in which is recorded the exact method for teaching each topic. The procedures are very specific. Dictatorships are not new, either educationally or politically. In both the Materials course and the Methods course it is quite evident that it is almost impossible to really divorce the content from the method. This nondivorcibility is perhaps the saving grace of both types of courses.

The Materials-Methods course, which is rather popular at the present time, is a recognition of the mutual dependence of the two types previously described. It has very great possibilities. In fact these possibilities have led to a point of view and to a more inclusive emphasis which we now call Professionalized Subject Matter. Just what is this Professionalized Subject Matter about which we are reading but which hasn't found its official way into many school catalogs? Perhaps the most recent and most elaborate treatment of the professional treatment of subject matter in arithmetic is that by Bond who considers the following items under each topic heading.[12]

1. Its historical background.
2. The extent of present use of the topic.
3. The method of applying the topic in everyday life.
4. Its grade position in the course of study.
5. The order of habit formation within it.
6. The most helpful literature including the most important investigations.
7. The psychology of arithmetic relating to it.
8. The analysis of the processes into their component elements.

[12] E. A. Bond, *The Professional Treatment of the Subject Matter of Arithmetic for Teacher Training Institutions Grades One to Six.*

It is especially interesting to note the inclusiveness of such a treatment. This movement has been under way for a decade stimulated by contributions in the literature by Professors Bagley, Upton and Randolph. Some subject matter people hope that this movement will check the rapid growth of courses in theoretical education and will lead to an increase of subject matter of a professional type. In the two year post high school course for teachers the prescribed mathematics in 1930 amounted to 2.8 per cent of the total while in 1905 the prescription amounted to 6.1 per cent of the total.[13]

In order to consider the three types of courses in the light of modern teacher training it will be necessary to discuss two more contributing factors: first, the desirable and probable trends in teacher training in general, and second, the desirable knowledge of a graduate of such a training institution in particular reference to arithmetic.

The first of these factors may appear to be speculative but it is not entirely so. For a number of years the magazines and yearbooks devoted to teacher training have been urging significant changes of point of view. Some of these tendencies that have a bearing upon arithmetic may be stated as below.

1. Not alone passive recognition but active acceptance of the principle that the laboratory school should occupy a significant and strategic central position in the work of teacher training. The real subjects in a teacher training institution are the normal school student and the laboratory school pupil.

2. General education courses, jointly and severally, are undergoing an evolution which will result in a selection and re-classification of the entire field. Critical thinking and evaluating as well as experimental factors instead of textbook tendencies will be dominant integrating factors.

3. Syllabi in the various courses in teacher training institutions are now being developed. They should recognize the weight of item one. The day of random lecturing from the fullness of one's experience or from the inspiration of the joke heard in the hallway seems to be rapidly passing.

4. In so far as possible and desirable every person engaged in the work of teacher training should have contact with the laboratory school. This contact perhaps should not be instructional in every case yet it is perhaps true that there are a number of really good teachers who can work on two levels of instruction.

5. In a democracy the teacher does and should occupy a significant position. Should any high school graduate be able to enter and usually be able to graduate from a teacher training institution? What factors make for teaching success? Can these be prognosticated? Answers to these questions are being sought and it is hoped will soon be more generally applied.

6. The teacher training institution is a part of the system of public educa-

[13] E. C. Class, *op. cit.*, p. 57.

tion and as such should not pursue a course of happy isolation. It ought to get into some sort of a cooperative alliance with the public schools.

The foregoing while not generally found in practice exist in isolated cases. The normal schools and teachers colleges are evolving.

Our public schools need new teachers who are not only able to go to work in the schools as now organized and constituted but also these new teachers should be able and willing to look ahead. The schools want teachers who have a rich and full background and who are willing to appreciate and understand children as individuals rather than as groups. These are general qualifications to which every course in teacher training should contribute. The professional course in arithmetic should make its contribution. This course in arithmetic should possess certain general aims or qualities which may be as given below.

1. A knowledge of the fundamentals of arithmetic. The fundamental ideas and concepts as well as the manipulative skills should be quite well understood. There are many good teachers who are able and willing to learn as they teach but if they do this they must have some basis upon which to build. The high school graduate's knowledge is too often confined to manipulation.

2. A background of material. It is desirable that the new teacher know something about the socio-economic field from which much of the arithmetic develops and it is important that she know where to get better acquainted with the quantitative world about her. The historical development of the use of certain phases of the subject and the modern use of these elements are useful in showing trends as well as in giving a bit of common culture concerning our race heritage.

3. A knowledge of educational principles. Despite the poor teaching techniques of some psychologists there are some very good principles of education and psychology that can be used in the organization of methods of teaching various topics in arithmetic. Arithmetic lends itself admirably to several schools of psychology. Perhaps the professional course in arithmetic should teach most of its own psychology.

4. A knowledge of techniques of teaching. The professional course should not resort to the static practice of prescribing one particular method for each topic. The teacher's personality might even be considered an element of method. Certainly it is a very strong factor in successful teaching. Perhaps new teachers should learn to use educational principles in adapting text and other materials. However, there may be danger in too much generalization with young teachers.

5. An appreciation of inherent philosophy. Who is willing to say that the only value in the study of arithmetic lies in the use to which contemporary society puts it? An understanding of the inherent potentialities of the various topics may be fruitful. Does the average citizen do enough figuring? It may be that our present economic status is partly due to our allowing the so-called experts to do both our figuring and thinking for us.

6. An acquaintance with materials of instruction. America is still very textbook conscious. In the past decade she has grown very syllabus, practice pad, and standard test conscious. These materials are all useful and the new teacher should know something about them and their use.

7. A knowledge and appreciation of pupil-teacher relation. The new teacher should see the materials and methods from a child's point of view as well as from the adult student's position. Much of this has to be gained by actual observation and participation in the laboratory school. The course in professional arithmetic should make a real educational use of the practice school.

8. A knowledge of researches and experiments. No doubt an active study of any subject and especially a professional study will raise many questions concerning content and method. Some of these questions have already been answered through research. The student should know something about these investigations and experiments and should be interested in seeking farther. The active open mind is desirable.

In formulating the eight aims of the professional arithmetic course the writer has had in mind a course which will help make elementary instruction a growing affair. Civilization is moving even if we do not at the present moment know in which direction. The schools have been built for the children and not for the teachers or the administrators.

The professional course in arithmetic should come just before the student teacher begins her work in the laboratory school. At present the course in arithmetic is most frequently prescribed in the first year of teacher training in courses of two, three, and four years length, respectively.[14]

In the chart below the writer has listed his eight main aims and

A Comparison of Three Types of Arithmetic Courses in Light of Eight Main Aims Theoretically Set up

The General Aims and Qualities	Subject Matter Course	Methods Course	Professionalized Sub. Matter
1. Mastery of ordinary arithmetic	A	D	C
2. Background of material	C−	D	C+
3. Educational principles	D	C+	C−
4. Techniques of teaching	D	A	C−
5. Inherent philosophy	C−	C−	C
6. Materials of instruction	D	C	C−
7. Child-education point of view	D	C+	C−
8. Researches and experiments	D	C	C

The chart should be interpreted according to the key: A=overemphasis, C=adequate, and D=deficient. Item one is read across as: in teaching mastery of ordinary arithmetic the Subject Matter Course gives overemphasis, the Methods Course is deficient, and the Professionalized Subject Matter Course is adequate.

[14] E. C. Class, *op. cit.*, p. 42.

qualities of the professional arithmetic course and has attempted to evaluate the three most commonly found types of course. It must be remembered that these are generalizations and that individual courses in particular schools differ from type and perhaps even vary from the catalog description of the course.

In general it seems fair to say that the Materials Course is the weakest of the three in respect to meeting all of the aims of the professional course in arithmetic and that the Professionalized Subject Matter Course is best adapted to the objectives as set up. It is also apparent that even the best of the three courses may or may not be adequate in its treatment of items 3, 4, 6, and 7. In order for any course to be really adequate in respect to all of the items it is essential that the instructor of the course holds a particular point of view with respect to his course and its relation to the whole task of progressive teacher training. In fact, the instructor must hold a growing philosophy of teacher training and be willing to work by it as well as speak of it. The philosophy that will adequately meet all of the objectives must in turn subscribe to the principle that the elementary school division and the college division shall live together continuously in spirit and at least occasionally in flesh. In order to show that the aims and doctrines presented previously are usable, a brief discussion of the treatment of some topic in arithmetic will be given. Some indicative questions will be presented and rather uncritically grouped. It is hoped however that sufficient organization is indicated so that it will be apparent that it is possible to attain a fulfillment of the eight main aims of the course.

Perhaps it is well to say at this time that it has been found necessary and desirable to review and reteach those principles of education and psychology which are very useful in discussing professional arithmetic. This topic has included certain pertinent educational concepts and has been treated in about two class periods. The topic comes early in the course so that it is usable as a guide in building up method for topics that follow. It may also be said that the writer favors a horizontal division of the material of arithmetic because that is the sequence followed in the elementary school (for which we train teachers) and also because such a division seems more natural and logical than does a vertical division. There appears to be a danger of repetition and even of omission in the vertical sectioning of the field of professional arithmetic. Conventional sequence however is not at all necessary.

For illustration let us take the topic of common fractions. This is an interesting topic to discuss because of the antiquity, the use, the disuse, and the curriculum argument that surround it. Suppose that we had previously treated whole numbers, an ordinary class would then assume that fractions should come next. This has not always been the case. Chauncey Lee in his American Accountant (1797) relates "The use of vulgar (common) fractions may be advantageously superseded by that of decimals, they are viewed as an unnecessary branch of common school education and are therefore omitted from this compendium." Several other texts of the same period did not treat fractions as a topic and included only a few very simple ones and we spend a full year in the elementary school upon this one topic. There are new books (perhaps modern) that have over one hundred pages of fraction material. Perhaps it is a good thing that children do not know all of these things about which educators have been arguing for many years.

Once we have decided to study the topic of fractions we might spend most or all of a class period raising questions and indicating methods of finding answers to our questions. Students learn by searching and finding but blind searching is not desirable. A first partial list of questions might include the following:

1. Why do fractions usually follow whole numbers?
2. Are fractions really in use in everyday life?
3. How much fraction material is found in current texts?
4. What effect have decimals had upon fractions?
5. Are fractions more prominent now than 50 and 100 years ago?
6. How did Egyptians, Babylonians, and Romans use fractions?
7. How much time does Erastus Root plan to devote to fractions?
8. What does Guy M. Wilson say about the use of fractions?
9. Will you teach fractions when you are placed in a public school?

Such questions are easily raised in a class and their study is quite different from the elementary treatment of the topic. These and similar questions contribute to the teacher's background and open up the the topic of investigation and research. With the realization that we still find a good deal of fraction material in our courses of study we can proceed to a study of skills and methods.

Another series of questions that deal with content and an approach to method may be illustrated by the following as a partial list.

1. What does a complete fraction concept involve?
2. What skills are required by the state course of study?

3. What skills are presented in some recent book?
4. Is text material similar to daily life material?
5. Can you write two good problems in each operation with fractions?
6. Why does inverting the divisor work? Can you give a proof?
7. Are there any axioms or general principles that apply to fractions?

The series of questions above leads to an analytic consideration of the material of fractions and this in turn involves the operations and for this reason, here is a fitting place for students to review and refresh concepts and skills. Here the writer suggests visiting the laboratory school where it has been found desirable to have a class working on a diagnostic exercise which has been prepared by some professional class in arithmetic.

The building of a diagnostic exercise in a topic such as fractions has been found very helpful. With guidance by the instructor, the exercises have usually been so constructed that they are quite usable in the laboratory school. Construction of such an exercise involves analysis of concepts and skills and the selection of material to measure these. It shows the importance of diagnosis and shows the new teacher how she may develop similar exercises for her use in teaching. The writer is convinced that some of the best as well as some of the worst teaching and testing materials are being constructed by present in-service teachers and that it is part of the work of the professional arithmetic course to give at least a beginning in this type of work. The diagnostic exercises completed by the pupils in the laboratory school might well be distributed among the students in the professional course so that these students may actually check them and prescribe remedial teaching. A class committee may even group the laboratory school class on the basis of the diagnosis and prescription and this grouping together with suggested procedures may be presented to the teacher in the laboratory school. Here again it would be well for the students to see the remedial procedures in the laboratory school. An exercise in fractions that was prepared by the professional class under the guidance of the writer appears below.

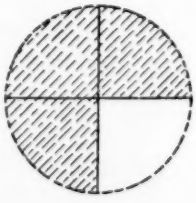
EXERCISE IN FRACTIONS

Name ..... Grade .....

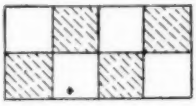
I. Understanding.

1. What part of the circle is shaded? .....
2. Group A is what part of group B? .....

A. ### B. #####



3. The rectangle is divided into ..... parts.
How many parts have been shaded? .....
What fractional part of the rectangle is shaded?
.....



4. Shade two-thirds of the rectangle given here.

5. Which is the larger: $\frac{11}{16}$ or $\frac{13}{16}$? .....
6. Which is the larger: $\frac{3}{7}$ or $\frac{3}{5}$? .....
7. Which is the larger: $\frac{5}{8}$ or $\frac{4}{6}$? .....

II. Reduction.

1. $\frac{3}{4} =$ /12
2. $\frac{15}{18} =$ 5/
3. $3\frac{1}{2} =$ /2
4. $\frac{27}{4} =$ .....

III. Addition

1. $\frac{1}{8} + \frac{2}{8} =$ .....
2. $\frac{2}{3} + \frac{5}{6} =$ .....
3. $\frac{2}{3}$ / $\frac{1}{6}$ / $\frac{5}{12}$
4. $\frac{1}{4}$ / $\frac{5}{6}$
5. $3\frac{1}{2}$ / $5\frac{1}{4}$
6. $2\frac{1}{2}$ / $3\frac{3}{4}$ / $6\frac{2}{3}$

IV. Subtraction

1. $\frac{3}{4} - \frac{1}{4} =$ .....
2. $\frac{5}{6} - \frac{1}{2} =$ .....
3. $\frac{5}{6} - \frac{2}{3} =$ .....
4. $26\frac{5}{8}$ / 14
5. $15\frac{5}{6}$ / $11\frac{1}{3}$
6. 27 / $8\frac{3}{4}$
7. $34\frac{1}{2}$ / $15\frac{3}{4}$

V. Multiplication

1. $\frac{1}{4}$ of 24 = ...
2. $\frac{2}{3} \times 21 =$ .....
3. $\frac{5}{7} \times 17 =$ .....
4. $22 \times \frac{5}{6} =$ .....
5. $18 \times 3\frac{2}{3} =$ .....
6. $\frac{5}{6} \times \frac{4}{15} =$ .....
7. $6\frac{2}{3} \times 4\frac{1}{2} =$ .....
8. $4\frac{3}{5} \times 3\frac{1}{3} =$ .....

VI. Division

1. $\frac{4}{5} \div 2 =$ .....
2. $\frac{6}{7} \div 5 =$ .....
3. $4 \div \frac{5}{6} =$ .....
4. $34 \div \frac{1}{5} =$ .....
5. $\frac{2}{3} \div \frac{5}{7} =$ .....
6. $\frac{5}{12} \div \frac{5}{8} =$ .....

Another series of questions that deal almost wholly with the development of method is illustrated by the following.

1. At what age or grade level do children first encounter the idea of fraction?

2. How shall we present the fraction concept?

3. Should a child have a complete fraction concept before proceeding to the work in calculation?

4. Why and how shall we teach reduction?

5. How shall we teach finding the common denominator?

6. What devices are in use in addition and subtraction? Are frames a real help in learning these operations?

7. Shall we have separate lessons on cancellation?

8. What gradations should be observed?

9. Should there be a denominator limit in schools?

10. Are the fractional cases worth teaching?

11. Shall we attempt to objectify all operations?

It may be desirable for the instructor to present at least some of the material as he would for an elementary class. He may even teach an elementary class with the professional class observing. In the development of method we have an excellent opportunity to use our principles of psychology.

Classes in teacher training institutions differ in intelligence, in previous mastery of materials, in professional interest, and in social and economic background. The professional course in arithmetic must shift emphasis upon phases of the work so that the class receives the most desirable training. Some classes have to be taught elementary materials almost the same as grade school classes. The medicine must always be suited to the ailment. The difficulty lies in the fact that usually one finds about twenty ailments among thirty or forty students. However these must be met and can be met by a flexible organization of a course which not only permits but requires individual students to assume some personal responsibility.

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On the Teaching of Algebra

By HOWARD D. GROSSMAN, *Dewitt Clinton High School, New York City*

THE IMPROVEMENT of the teaching of mathematics is the proper domain of teachers as well as of supervisors. It is on that assumption that the following remarks are made.

The syllabus in Elementary Algebra issued by the University of the State of New York in 1930 can command in every teacher of that subject the most whole-hearted admiration and desire to co-operate. Even the most radical can find little to censure; can only hope in fact that in other fields (like economics) more dominated by irrational prejudices and selfish motives, such rational syllabi could be drawn up.

The authors of the syllabus however deplore the extent to which practice has lagged, and is still lagging, behind such admirable theory. I wonder if the discrepancy might not be lessened not so much by a frontal attack on teaching as on some of the more important influences on teaching, viz., supervision, teacher-rating, school syllabi to teachers, regents' and school uniform tests, and text-books.

If, as the state syllabus lays down, the chief end in the teaching of algebra is the teaching of the function-concept, supervisors should focus attention on this and teacher-efficiency should be judged largely by the extent to which the function-concept is taught. Supervisors should select the significant elements (significant from the point of view of the state syllabus) in teaching rather than the petty and irrelevant details; to this end it might be better for a supervisor to select for observation several shorter cross-sections of a teacher's periods than one longer one.

Teacher-rating, it naturally follows, should be largely a mark of the degree to which the function-concept is taught, to which the power of generalization, of deriving formulas is developed, and to which the formula, equation, and graph are correlated.

It is further undeniable that school syllabi to teachers influence teaching and so they should be drawn up very carefully in accord with the principles of the state syllabus; there should be included special references to co-ordination between topics, which a syllabus

by its very nature tends to ignore and which is stressed as an objective in the state syllabus. It is very well to tell teachers themselves to co-ordinate the topics in the school syllabi but it is much better to make some explicit provision in the syllabi for such co-ordination.

Regents' examinations give only one optional formula-question and two short function-questions the answers to which could be easily coached without teaching much of the function-concept. School uniform tests are even less adequate. A variety of subtler questions involving the function-concept can and must be devised; perhaps some early questions in the calculus, like questions about the maxima and minima of given curves, etc., would do.

Finally, we talk so much about formulas yet our text-books have only a few meager formulas about rectangles, circles, etc. With a little research in the statistics of economics, psychology, eugenics, biology, etc., especially in the correlations between simple related phenomena like heights of father and son, we could compile a really staggering set of simple approximate formulas to constitute the stock-in-trade of all algebra teachers.

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